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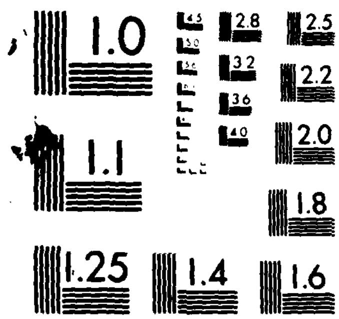

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CORRECTING FOR TURBULENCE EFFECTS ON AVERAGE VELOCITY MEASUREMENTS MADE USING FIVE HOLE SPHERICAL PITOT TUBE PROBES

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

CORRECTING FOR TURBULENCE EFFECTS ON AVERAGE VELOCITY MEASUREMENTS MADE USING FIVE HOLE SPHERICAL PITOT TUBE PROBES

by

Robert D. Pierce

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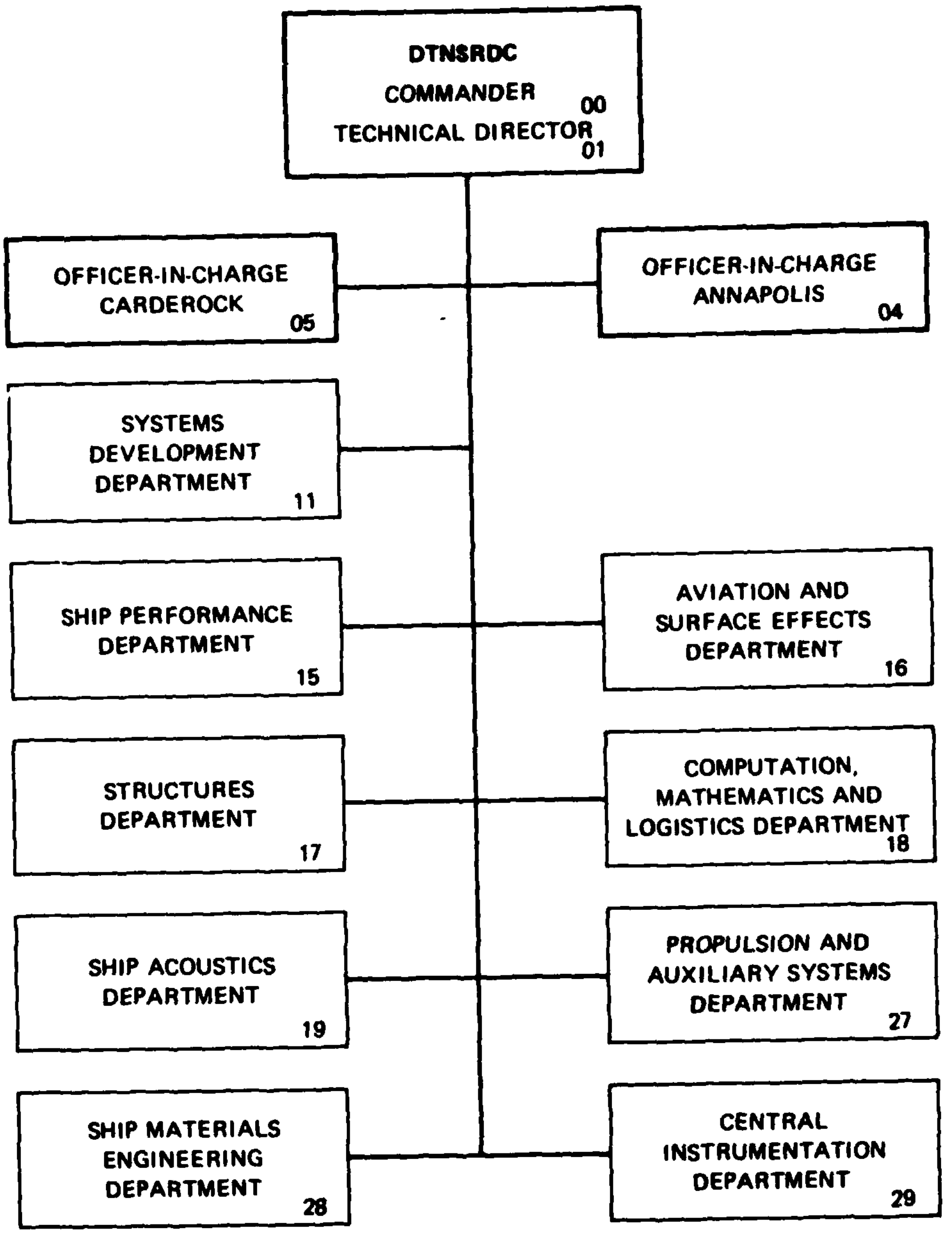

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A procedure has been developed for correcting five hole spherical pitot tube mean velocity measurements for errors caused by velocity fluctuations associated with turbulent flow. Velocity fluctuations are independently measured with hot film or hot wire anemometers. These data are then applied to expressions derived in this report to accomplish the necessary correction. This procedure applies to the case where the pitot tube responds to average

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pressure differentials and is not oriented to null the cross velocity components. For the special case of isotropic flow, the results derived in this report show that corrections are not required.

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LIST OF ABBREVIATIONS

C_{UxUy}, C_{UxUz}, C_{UyUz}	Covariance between velocity components
DTNSRDC	David W. Taylor Naval Ship Research and Development Center
$E[\cdot]$	Expected value operator
K_n	Intermediate quantities that relate measured pressure and hole separation terms to the flow velocity components
$\overline{K}_n$	Averaged or expected value of the K_n terms
$\overline{K}_{nc}$	The $\overline{K}_n$ terms corrected for velocity fluctuations
p	Pressure at a point on the sphere
p_o	Pressure of the undisturbed flow
Q_{n1}	Intermediate quantities that relate pressure differences between the first and the n-th hole to the fluid velocity terms
$\vec{U}$	Fluid velocity vector
U	Magnitude of the fluid velocity
Ux, Uy, Uz	Flow velocity components

LIST OF ABBREVIATIONS (Cont.)

$\overline{U}x$, $\overline{U}y$, $\overline{U}z$	Average flow velocity components
$b_{\overline{U}x}$, $b_{\overline{U}y}$, $b_{\overline{U}z}$	Average flow velocity components biased by dynamic pressure head
w	Weight per unit volume of the fluid
x, y, z	Cartesian coordinates

ρ	Magnitude	Polar space coordinates
α	Angle to x axis	
β	Angle to y axis	
γ	Angle to z axis	

LIST OF ABBREVIATIONS (Cont.)

α_o, β_o, γ_o	Polar space coordinates for flow velocity direction
α_s	Separation angle between holes on the sphere
θ	Angle between the stagnation point and a specified point on the sphere
θ_{np}	Angle between stagnation point and the n-th pitot tube hole
σ^2_{Ux}, σ^2_{Uy}, σ^2_{Uz}	Variance of velocity components

ABSTRACT

A procedure has been developed for correcting five hole spherical pitot tube mean velocity measurements for errors caused by velocity fluctuations associated with large scale turbulent flow. Velocity fluctuations are independently measured with hot film or hot wire anemometers. These data are then applied to expressions derived in this report to accomplish the necessary correction. This procedure applies to the case where the pitot tube responds to average pressure differentials and is not oriented to null the cross velocity components. For the special case of isotropic flow, the results derived in this report show that corrections are not required.

ADMINISTRATIVE INFORMATION

This work was sponsored by the Large Scale Vehicle (LSV) program under Task SF43400391 and Element 62543N (Job order 1-1946-161). This program includes the development of a wake rake to measure the wake field behind this model. The wake measurements will include the use of pitot tubes and correction procedures for these measurements are planned. These correction procedures will also be applied to experiments performed at DTNSRDC in support of the LSV program.

INTRODUCTION

Five hole pitot tubes provide a simple, reliable means for measuring the magnitude and direction of fluid velocity. At the David W. Taylor Naval Ship Research and Development Center (DTNSRDC), these probes are used in air or water in model basin, water tunnel or wind tunnel experiments. In these applications, the probes are located in a fixed position; they are not adjusted to null the cross velocity components. Accurate measurements of the three dimensional velocity components can be made when experiments are performed in low turbulence flow fields. In certain flow fields, however, such as the thick wake behind a model, inaccuracies occur in these measurements due to turbulence effects and flow shear effects. This report presents a method for correcting the measurements for turbulence effects when hot film or hot wire anemometer data are available. Inaccuracies due to shear flow are not considered.

The velocity fluctuations present in turbulent flow affect pitot tube measurements of the mean velocity in the following manner. The fluid flow over the surface of the pitot tube produces a pressure distribution that varies spatially over the probe. For a five hole pitot tube, these pressures are measured at five locations (or holes) on the face of the probe (one inner hole and four outer holes); pressure differences between the outer holes and the inner hole are obtained and converted into velocity measurements. Instantaneous pressure measurements at these locations are desired; however, pitot tubes must be small to avoid significant disturbance of the flow field and to minimize the measuring volume. Highly accurate pressure transducers are large, so tubes are used to transmit these pressures to remote pressure transducers. These tubes have the effect of filtering or averaging the pressure measurement regardless of the frequency response characteristics of the pressure transducers and other related signal conditioning. Pressure, however, is proportional to velocity squared, so average pressure consists of two components when velocity fluctuations are present: the average pressure associated with the mean velocity and a dynamically induced pressure head component associated with the average of the squared velocity fluctuations. When pressure differences are taken, these dynamically induced pressure head terms do not always completely cancel. The mean velocity measurements obtained from these pressure differentials will then contain an error contributed by the average of the squared velocity fluctuations.

These velocity measurements can be corrected for the effects of filtered pressure fluctuations if hot film or hot wire anemomenter data are available. Hot film and hot wire anemometers have excellent high frequency response characteristics; however, they tend to drift. Average velocity measurements from these sensors are not reliable unless unusually careful measurements are made. They can, however, provide the velocity fluctuation data necessary to correct the pitot tube average velocity measurements. To accomplish this, two data collection runs are necessary at the same location -- one for the pitot tube and one for the hot film or hot wire anemometer. For a circular region, this dual measurement may be made by placing the probes 180 degrees apart at the same radius on a rotatable support; measurements are made simultaneously as the probes are indexed over 360 degrees.

BACKGROUND AND ASSUMPTIONS

The effect of turbulence on pressure probe measurements has been studied for many years; in each case, however, the probe is assumed to be aligned in at least one plane of the direction of the mean flow (null reading mode). Goldstein[1]* in 1936, for example, proposed a turbulence correction procedure for a static pressure probe in isotropic turbulence. More recently in 1977, Bennett[2] presented a method for correcting turbulence effects in aligned five hole pitot tubes. Further, Christianson and Bradshaw[3] in 1981 described the effects of turbulence on various types of pressure probes; one of their conclusions is that these devices should always be used in a null reading mode for turbulent flow measurements.

The use of these probes as null reading devices is not always possible. The measurement of the three dimensional wake in a propeller plane, for example, is a case where an array of these probes are positioned at different radii in the propeller plane;

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* A complete listing of references is given on page 13.

individual adjustment of each probe is impractical since readjustment is required for each angular position of this array. Space limitations and other considerations are cited by Treaster and Yocum[4] for their application of five hole probes in a non-nulling mode.

To allow the use of five hole probes in a non-nulling mode, a procedure is developed to correct the mean velocity measurements for the effects of turbulence. This correction procedure is based on a number of assumptions about the probe geometry, probe calibration and the flow field. First, spherical five hole pitot tubes are usually used for these measurements, so this form of probe geometry is assumed. For a particular probe, calibrations are performed to relate velocity to the pressure differences across pairs of holes on the probe. The correction procedure, however, assumes the pressure distribution associated with the spherical form, so the assumption is made that the calibration closely matches the theoretical distribution. Differences between calibration and theoretical results are second order effects, such as: inviscid fluid assumed in the theoretical derivation, slight differences in the spherical shape of the actual probe, finite diameter holes on the sphere for measuring the local pressure, and misalignment of these holes. Although the pitot tube measurements are made separately from the hot film or hot wire anemometer measurements, the assumption is made that they are placed in the same location and that the flow conditions are the same for each case. Further, the velocity fluctuations associated with the turbulent flow field are assumed to be stationary, ergodic random processes.

Also, assumptions about the velocity gradients and the sizes of the turbulence eddies in the flow field are required. These assumptions are made whenever pitot tube and hot film or hot wire anemometers are used; they are not unique to the correction procedure presented in this report. The velocity gradients are assumed to be small and the sizes of the turbulence eddies are assumed to be large compared to the pitot tube diameter and the measuring volume of the hot film or hot wire anemometer; over this region the sensors see a quasi-stationary uniform flow. For the pitot tube, this assumption is comparable to simulating the velocity fluctuations and pressure measurements on a sphere by rotating the sphere in an inviscid flow field; the angular motion about a mean angle of attack simulates directional changes in velocity.

The range of turbulence intensities over which the correction procedure is applicable is also restricted. If the angle of attack is too great, then viscosity effects in the boundary layer will change the pressure distribution at the points on the sphere where the pressure is measured. From Pien[5], the region on the sphere that is within about 40 degrees of the stagnation point is nearly independent of Reynold's number. This result is used to establish a criterion for the largest instantaneous angle of attack assumed to exist in the flow field. Given that the angular distance between the center hole and the side holes is 25 degrees, for example, the instantaneous angle of attack is restricted to be within +15 or -15 degrees of the center hole.

CORRECTING MEAN VELOCITY MEASUREMENTS FOR VELOCITY FLUCTUATIONS

The procedure for correcting the mean velocity measurements for velocity fluctuations is based on the theoretical relationship between velocity and the pressure on the sphere. Pien[5] was the first to show that the velocity component in any plane is obtained independently from three pressure measurements in that plane. A slightly different derivation of these expressions is presented in Appendix A; this derivation is included since it gives better insight into the correction procedure. An example of this correction procedure is given in Appendix B.

From Appendix A, equations (A.29) through (A.32) relate the measured quantities (pressure, hole separation angle and physical constants) to the three velocity components (Ux, Uy, Uz). These equations are

$$K_1 = \frac{Q_{51} - Q_{21}}{4 \sin\alpha_s \cos\alpha_s} = Ux\ Uy \qquad (1)$$

$$K_2 = \frac{Q_{51} + Q_{21}}{2 \sin^2\alpha_s} = Uy^2 - Ux^2 \qquad (2)$$

$$K_3 = \frac{Q_{41} - Q_{31}}{4 \sin\alpha_s \cos\alpha_s} = Ux\ Uz \qquad (3)$$

$$K_4 = \frac{Q_{41} + Q_{31}}{2 \sin^2\alpha_s} = Uz^2 - Ux^2 \qquad (4)$$

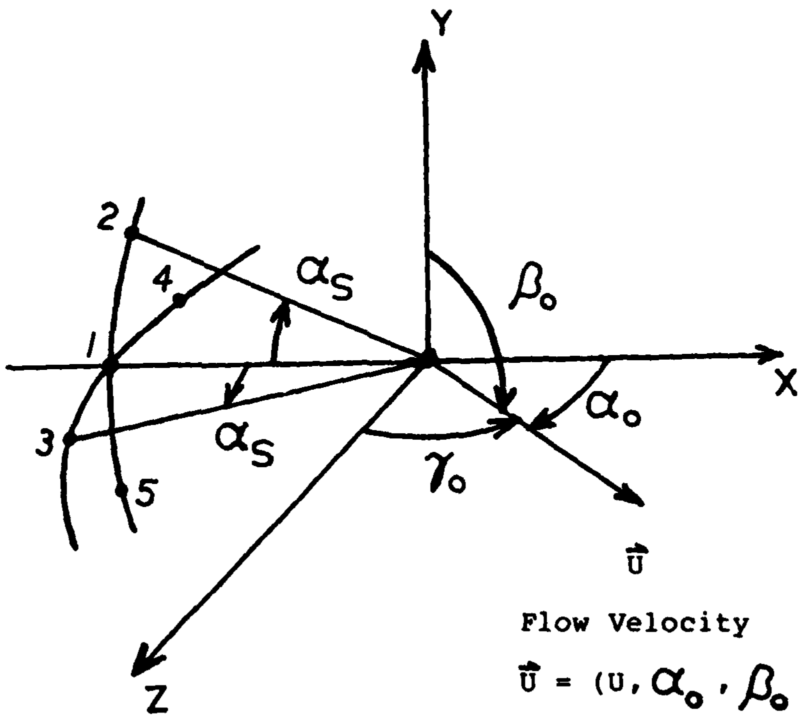
where the K_n terms summarize the measured quantities; α_s is the hole separation angle; and the Q_{n1} terms (See equation (A.13).) contain the physical constants and the pressure differences between the n-th side hole and the center hole. The hole locations and the coordinate system are described in Figure 1.

The K_n terms in these expressions are proportional to the measured pressure that is transmitted by tubes to the pressure transducers. Since the velocity fluctuations are assumed to be stationary, ergodic random processes, the filtering or averaging performed by these tubes is idealized by the expected value

Holes 1, 2 and 5 are in the xy plane.
Holes 1, 3 and 4 are in the xz plane.

Hole
Locations

Hole Separation
angle is α_S .



Flow Velocity

$\vec{U} = (U, \alpha_o, \beta_o, \gamma_o)$

where $U = |\vec{U}|$

Conversion from polar space
to cartesian coordinates:

$$x = \rho \cos\alpha$$
$$y = \rho \cos\beta$$
$$z = \rho \cos\gamma$$

where $\rho = (x^2 + y^2 + z^2)^{1/2}$

Figure 1 - Pitot Tube Hole Locations and Polar
Space Coordinate System

operator, E[·]. This operator will also include any other filtering or averaging that is generally done during acquisition and analysis of this pressure data. The expected or mean value of these expressions are

$$E[K_1] = \bar{K}_1 = \bar{U}x\ \bar{U}y + C_{UxUy} \tag{5}$$

$$E[K_2] = \bar{K}_2 = \bar{U}y^2 - \bar{U}x^2 + \sigma^2_{Uy} - \sigma^2_{Ux} \tag{6}$$

$$E[K_3] = \bar{K}_3 = \bar{U}x\ \bar{U}z + C_{UxUz} \tag{7}$$

$$E[K_4] = \bar{K}_4 = \bar{U}z^2 - \bar{U}x^2 + \sigma^2_{Uz} - \sigma^2_{Ux} \tag{8}$$

where the $\bar{K}_n$ terms are the mean K_n terms; $\bar{U}x$, $\bar{U}y$ and $\bar{U}z$ are the mean velocity components; and the variance and covariance of the velocity components are given by the σ and C terms. The variance and covariance terms in these expressions represent the dynamically induced pressure head. The variance (standard deviation squared) of each velocity component is: σ^2_{Ux} for Ux, σ^2_{Uy} for Uy and σ^2_{Uz} for Uz. The covariance for Ux and Uy is C_{UxUy}, and it is C_{UxUz} for Ux and Uz.

The expected value operation that produced these expressions does not require an assumed form or distribution for the probability density function of the velocity fluctuations. In practice, however, the mean values, variances and covariances are estimated from finite length data records, and random error is associated with these estimates. A theoretical description of this random error would require an assumed distribution; see Bendat and Piersol[6].

Note that the special case of isotropic turbulent flow does not require corrections. For this form of turbulent flow, the three velocity variance terms are equal and the covariance terms are zero (See Hinze[7].) In equations (5) and (7), the covariance terms are zero; in equations (6) and (8), the variance terms cancel each other. Hence, the dynamic pressure head terms do not contribute to the average pressure measurements in equations (5) through (8), so the mean velocity measurements are not biased by the effects of isotropic turbulence.

The velocity fluctuation correction procedure first requires measurement of the $\overline{K}_n$ terms. These averaged pressures are used with the uniform (steady) flow calibration to extract mean velocity measurements that contain the effect of the dynamic pressure head. This calibration is generally performed using the procedures documented by Pien[5]. Since Ux is taken to be in both planes of the pitot tube hole array, two values of Ux are obtained, one for the xy plane and one for the xz plane. For the measured velocity components from the xy plane,

$$\overline{K}_1 = {}^{b}\overline{U}x \;\; {}^{b}\overline{U}y \tag{9}$$

$$\overline{K}_2 = {}^{b}\overline{U}y^2 - {}^{b}\overline{U}x^2 \tag{10}$$

And, from the xz plane components,

$$\overline{K}_3 = {}^{b}\overline{U}x \;\; {}^{b}\overline{U}z \tag{11}$$

$$\overline{K}_4 = {}^{b}\overline{U}z^2 - {}^{b}\overline{U}x^2 \tag{12}$$

where the superscript, b, indicates that these velocity measurements are biased by the dynamic pressure head.

In the second part of the correction procedure, corrected values for the $\bar{K}_n$ terms, $\bar{K}_{nc}$, are obtained by rewriting equations (5) through (8) as follows

$$\bar{K}_{1c} = \bar{K}_1 - C_{UxUy} = \bar{U}x \quad \bar{U}y \tag{13}$$

$$\bar{K}_{2c} = \bar{K}_2 - \sigma^2_{Uy} + \sigma^2_{Ux} = \bar{U}y^2 - \bar{U}x^2 \tag{14}$$

$$\bar{K}_{3c} = \bar{K}_3 - C_{UxUz} = \bar{U}x \quad \bar{U}z \tag{15}$$

$$\bar{K}_{4c} = \bar{K}_4 - \sigma^2_{Uz} + \sigma^2_{Ux} = \bar{U}z^2 - \bar{U}x^2 \tag{16}$$

The variance and covariance terms are obtained from measurements using a hot film or hot wire anemometer. This anemometer is placed in the same location as was the pitot tube; the flow conditions must be the same when each probe acquires data.

Once the $\bar{K}_{nc}$ terms are determined, the corrected velocity measurements are obtained from equations (13) through (16) by assuming that $\bar{U}x$ is known to be positive and larger than $\bar{U}y$ and $\bar{U}z$. For the xy plane (equations (13) and (14)),

$$\bar{U}x = \left\{ \frac{-\bar{K}_{2c} + \left(\bar{K}_{2c}^{\,2} + 4\ \bar{K}_{1c}^{\,2} \right)^{\frac{1}{2}}}{2} \right\}^{\frac{1}{2}} \tag{17}$$

$$\overline{U}y = \frac{\overline{K}_{1c}}{\overline{U}x} \qquad (18)$$

and for the xz plane (equations (15) and (16)),

$$\overline{U}x = \left\{ \frac{-\overline{K}_{4c} + \left(\overline{K}_{4c}^{\,2} + 4\,\overline{K}_{3c}^{\,2}\right)^{\frac{1}{2}}}{2} \right\}^{\frac{1}{2}} \qquad (19)$$

$$\overline{U}z = \frac{\overline{K}_{3c}}{\overline{U}x} \qquad (20)$$

Two answers for $\overline{U}x$ are obtained. Differences between these two values are attributable to many factors, such as: transducer errors; estimation error for the mean, variance or covariance; viscous effects due to large turbulent intensities; shear flow; and failure of the calibration to match the theoretical relationship between velocity and pressure.

CONCLUSIONS

For the given set of assumptions about the pitot tube geometry and the flow conditions, five hole pitot tube mean velocity measurements can be corrected for the velocity fluctuations associated with turbulent flow using hot film or hot wire anemometer data. Also, isotropic turbulent flow does not affect these mean velocity measurements, so corrections are not necessary for this form of turbulence.

REFERENCES

1. Goldstein, S., "A Note on the Measurement of Total Head and Static Pressure in a Turbulent Stream", Proc. Royal Soc. A, Vol. CLV, pg. 570, (1936).

2. Bennett, J.C., "Use of Five-Hole Pneumatic Probe in Unsteady Flows", Experimental Diagnositics in Gas Phase Combustion Systems, Volume 53 Progress in Astronautics and Aeronautics, Edited by M. Summerfield, Princeton University, Princeton, New Jersey, (1977).

3. Christianson, T. and P. Bradshaw, "Effect of Turbulence on Pressure Probes", Journal of Physics E, Vol. 14 No. 8., pg 992, (Aug. 1981).

4. Treaster, A. and A. Yocum, "The Calibration and Application of Five-hole Probes", ISA Transactions, Vol. 18 No. 3, pg 23, (1979).

5. Pien, P.C., "Five-Hole Spherical Pitot Tube", DTMB Report No. 1229, (May 1958).

6. Bendat, J. and A. Piersol, Random Data: Analysis and Measurement Procedures, Wiley-Interscience, N.Y., N.Y., (1971).

7. Hinze, J.O., Turbulence, McGraw-Hill, N.Y.,N.Y., (1975).

APPENDIX A

VELOCITY MEASUREMENTS FROM A FIVE HOLE SPHERICAL PITOT TUBE

A five hole spherical pitot tube uses the pressure distribution about a sphere in uniform flow to measure velocity. The pressure at each hole on the sphere is measured and these measurements are then converted into the three velocity components. The correction procedure for velocity fluctuations is based on this relationship so these equations are derived.

Following Pien[5], the pressure at any point on a sphere is given by

$$\frac{p - p_o}{w} = \frac{U^2}{2g}\left(1 - \frac{9}{4}\sin^2\theta\right) \tag{A.1}$$

where

U - magnitude of the fluid velocity

p - pressure at a point on the sphere

p_o - pressure of the undisturbed flow

w - weight per unit volume of the fluid

θ - angle between the stagnation point and a specified point on the sphere

g - gravitational constant

For a sphere, this pressure relationship is independent of the diameter of the sphere. This pressure relationship is rewritten as

$$\frac{8\,g}{w}\,(\,p - p_o\,) = U^2\,(\,9\cos^2\theta - 5\,) \qquad \text{(A.2)}$$

The hole locations and the polar space coordinate system are described in Figure 1. When vectors are specified to different hole locations, a radius of one is assumed for the sphere; generality is preserved since pressure is independent of sphere diameter.

The angles between the stagnation point and each hole are required. So vectors are defined to each of these points, and the vector dot product gives the cosine of the angle between these points. For both polar space and cartesian coordinates, the location of each hole is:

HOLE NUMBER	POLAR SPACE $(\rho, \alpha, \beta, \gamma)$	CARTESIAN COORDINATES (x,y,z)
1	$(1, \pi, \frac{\pi}{2}, \frac{\pi}{2})$	$(-1, 0, 0)$
2	$(1, \pi - \alpha_s, \frac{\pi}{2} - \alpha_s, \frac{\pi}{2})$	$(-\cos\alpha_s, \sin\alpha_s, 0)$
3	$(1, \pi - \alpha_s, \frac{\pi}{2}, \frac{\pi}{2} - \alpha_s)$	$(-\cos\alpha_s, 0, \sin\alpha_s)$
4	$(1, \pi - \alpha_s, \frac{\pi}{2}, \frac{\pi}{2} + \alpha_s)$	$(-\cos\alpha_s, 0, -\sin\alpha_s)$
5	$(1, \pi - \alpha_s, \frac{\pi}{2} + \alpha_s, \frac{\pi}{2})$	$(-\cos\alpha_s, -\sin\alpha_s, 0)$

where α_s is the separation angle between holes. The upstream stagnation point is located at:

POLAR SPACE	CARTESIAN COORDINATES
$(1, \pi - \alpha_o, \pi - \beta_o, \pi - \gamma_o)$	$(-\cos\alpha_o, -\cos\beta_o, -\cos\gamma_o)$

The stagnation point vector is a unit vector in the direction opposite to the flow velocity vector. Using the dot product between each hole location vector and the stagnation point, the cosine of the angle between each hole and the stagnation point is

$$\cos\theta_{1p} = \cos\alpha_o \tag{A.3}$$

$$\cos\theta_{2p} = \cos\alpha_s\cos\alpha_o - \sin\alpha_s\cos\beta_o \tag{A.4}$$

$$\cos\theta_{3p} = \cos\alpha_s\cos\alpha_o - \sin\alpha_s\cos\gamma_o \tag{A.5}$$

$$\cos\theta_{4p} = \cos\alpha_s\cos\alpha_o + \sin\alpha_s\cos\gamma_o \tag{A.6}$$

$$\cos\theta_{5p} = \cos\alpha_s\cos\alpha_o + \sin\alpha_s\cos\beta_o \tag{A.7}$$

Since the cosine squared is required for the pressure distribution equation (equation (A.2)) then

$$\cos^2\theta_{1p} = \cos^2\alpha_o \tag{A.8}$$

$$\cos^2\theta_{2p} = \cos^2\alpha_s\cos^2\alpha_o + \sin^2\alpha_s\cos^2\beta_o - 2\sin\alpha_s\cos\alpha_s\cos\alpha_o\cos\beta_o \tag{A.9}$$

$$\cos^2\theta_{3p} = \cos^2\alpha_s\cos^2\alpha_o + \sin^2\alpha_s\cos^2\gamma_o - 2\sin\alpha_s\cos\alpha_s\cos\alpha_o\cos\gamma_o \tag{A.10}$$

$$\cos^2\theta_{4p} = \cos^2\alpha_s\cos^2\alpha_o + \sin^2\alpha_s\cos^2\gamma_o + 2\sin\alpha_s\cos\alpha_s\cos\alpha_o\cos\gamma_o \tag{A.11}$$

$$\cos^2\theta_{5p} = \cos^2\alpha_s\cos^2\alpha_o + \sin^2\alpha_s\cos^2\beta_o + 2\sin\alpha_s\cos\alpha_s\cos\alpha_o\cos\beta_o \tag{A.12}$$

To eliminate p_o (pressure of the undisturbed flow), the pressure difference between each outer hole and the center hole is defined. Let

$$Q_{n1} \equiv \frac{8\ g}{9\ w}(p_n - p_1) = U^2(\cos^2\theta_{np} - \cos^2\theta_{1p}) \tag{A.13}$$

So

$$Q_{21} = U^2 (\cos^2 \theta_{2p} - \cos^2 \theta_{1p}) \quad (A.14)$$

$$Q_{31} = U^2 (\cos^2 \theta_{3p} - \cos^2 \theta_{1p}) \quad (A.15)$$

$$Q_{41} = U^2 (\cos^2 \theta_{4p} - \cos^2 \theta_{1p}) \quad (A.16)$$

$$Q_{51} = U^2 (\cos^2 \theta_{5p} - \cos^2 \theta_{1p}) \quad (A.17)$$

Substituting the angle relationships given in equations (A.8) through (A.12), then

$$Q_{21} = \sin^2\alpha_s(-Ux^2 + Uy^2) - 2 \sin\alpha_s \cos\alpha_s \; Ux \; Uy \quad (A.18)$$

$$Q_{31} = \sin^2\alpha_s(-Ux^2 + Uz^2) - 2 \sin\alpha_s \cos\alpha_s \; Ux \; Uz \quad (A.19)$$

$$Q_{41} = \sin^2\alpha_s(-Ux^2 + Uz^2) + 2 \sin\alpha_s \cos\alpha_s \; Ux \; Uz \quad (A.20)$$

$$Q_{51} = \sin^2\alpha_s(-Ux^2 + Uy^2) + 2 \sin\alpha_s \cos\alpha_s \; Ux \; Uy \quad (A.21)$$

Where the three components of velocity in cartesian coordinates are given by

$$Ux = U \cos\alpha_o \tag{A.22}$$

$$Uy = U \cos\beta_o \tag{A.23}$$

$$Uz = U \cos\gamma_o \tag{A.24}$$

Taking the sum and difference of the Q_{n1} terms in the xy plane gives

$$Q_{51} - Q_{21} = 4 \sin\alpha_s \cos\alpha_s \; Ux \; Uy \tag{A.25}$$

$$Q_{51} + Q_{21} = 2 \sin^2\alpha_s \left(- Ux^2 + Uy^2 \right) \tag{A.26}$$

and similarly for the xz plane

$$Q_{41} - Q_{31} = 4 \sin\alpha_s \cos\alpha_s \; Ux \; Uz \tag{A.27}$$

$$Q_{41} + Q_{31} = 2 \sin^2\alpha_s \left(- Ux^2 + Uz^2 \right) \tag{A.28}$$

By defining K_n , the measured and desired quantities are separated.

$$\underbrace{K_1 \equiv \frac{Q_{51} - Q_{21}}{4 \sin\alpha_s \cos\alpha_s}}_{\text{MEASURED QUANTITIES}} = \underbrace{Ux\ Uy}_{\text{DESIRED QUANTITIES}} \tag{A.29}$$

$$K_2 \equiv \frac{Q_{51} + Q_{21}}{2 \sin^2\alpha_s} = Uy^2 - Ux^2 \tag{A.30}$$

$$K_3 \equiv \frac{Q_{41} - Q_{31}}{4 \sin\alpha_s \cos\alpha_s} = Ux\ Uz \tag{A.31}$$

$$K_4 \equiv \frac{Q_{41} + Q_{31}}{2 \sin^2\alpha_s} = Uz^2 - Ux^2 \tag{A.32}$$

MEASURED QUANTITIES　　DESIRED QUANTITIES

The results presented by Pien[5] can be obtained by projecting the velocity vector onto the xy and xz planes. The magnitudes and angles of these projections are then related to the K_n terms presented in equations (A.29) through (A.32).

APPENDIX B
AN EXAMPLE OF THE VELOCITY FLUCTUATION CORRECTION PROCEDURE

From pitot tube measurements, the following biased velocity measurements were obtained in the xy plane

$$b_{\overline{U}x} = 0.445 \qquad b_{\overline{U}y} = -0.00962$$

and in the xz plane

$$b_{\overline{U}x} = 0.447 \qquad b_{\overline{U}z} = -0.0298$$

(All velocity measurements are normalized by the free stream velocity.) From hot film anemometer measurements, the following variance and covariance measurements for the three velocity components were obtained:

$$\sigma^2_{Ux} = 0.00360 \qquad C_{UxUy} = 0.00191$$

$$\sigma^2_{Uy} = 0.00628 \qquad C_{UxUz} = -0.00191$$

$$\sigma^2_{Uz} = 0.00469 \qquad C_{UyUz} = -0.00197$$

The turbulence intensity (total velocity standard deviation divided by total mean velocity) is 0.27. The $\overline{K}_n$ terms from the pitot tube data are given for the xy plane using equations (9) and (10)

$$\overline{K}_1 = (0.445)(-0.00962) = -0.00428$$

$$\overline{K}_2 = (-0.00962)^2 - (0.445)^2 = -0.1979$$

and for the xz plane using equations (11) and (12)

$$\bar{K}_3 = (0.447)(-0.0298) = -0.01332$$

$$\bar{K}_4 = (-0.0298)^2 - (0.447)^2 = -0.1989$$

The corrected $\bar{K}_n$ terms, $\bar{K}_{nc}$, are given by equations (13) through (16)

$$\bar{K}_{1c} = -0.00428 - 0.00191 = -0.00619$$

$$\bar{K}_{2c} = -0.1979 - 0.00628 + 0.00360 = -0.2006$$

$$\bar{K}_{3c} = -0.01332 + 0.00191 = -0.01141$$

$$\bar{K}_{4c} = -0.1989 - 0.00469 + 0.00360 = -0.2000$$

Using equations (17) and (18), the corrected $\bar{U}x$ and $\bar{U}y$ terms for the xy plane are

$$\bar{U}x = 0.4481$$

$$\bar{U}y = -0.0138$$

and from equations (19) and (20), the corrected $\bar{U}x$ and $\bar{U}z$ terms for the xz plane are

$$\bar{U}x = 0.4480$$

$$\bar{U}z = -0.0255$$

For either the xy or xz planes, $\bar{U}x$ is 0.448. In percent of reading, $\bar{U}x$ from the xy plane changed by 0.7% and $\bar{U}x$ from the xz plane changed by 0.2%. In percent of total velocity at the measurement location, $\overline{U}y$ changed by 0.9% and $\overline{U}z$ changed by 1.0%. In percent of reading, $\overline{U}y$ changed by 30% and $\overline{U}z$ changed by 17%.

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